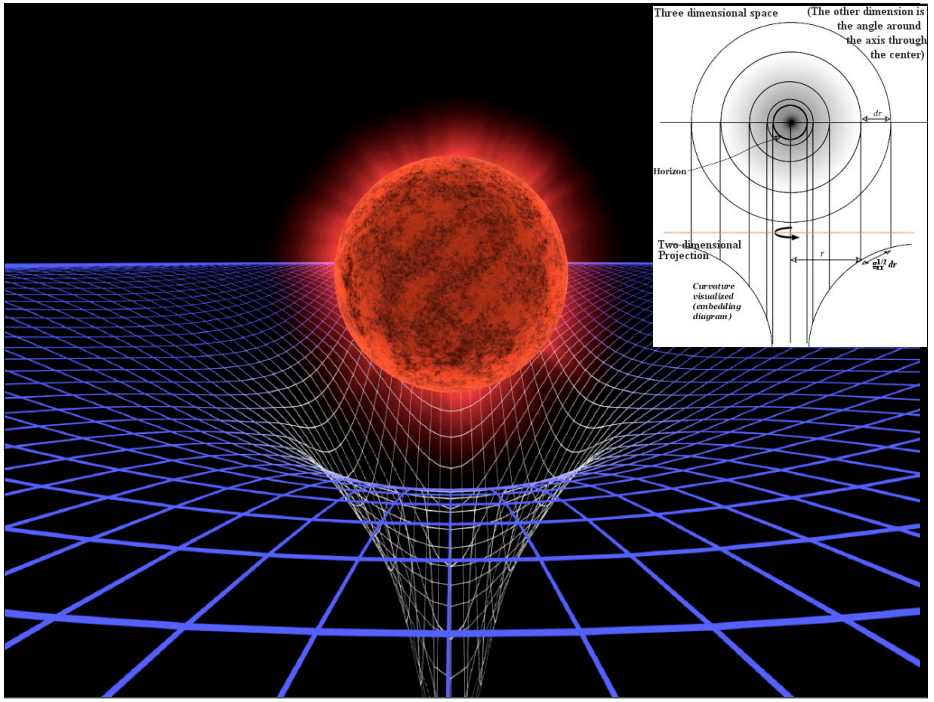
Consequences of the Equivalence Principle

$$\gamma_G = \frac{1}{\sqrt{1 - r_s/d}} \qquad r_s = \frac{2GM}{c^2} = 3 \text{ km} \left(\frac{M}{M_\odot}\right)$$

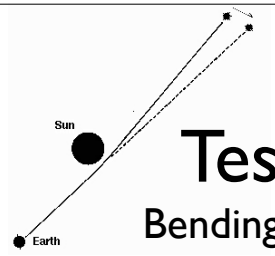
- Gravity bends light = warps space-time by factor γ
- time slows down where gravity is strong
  - $t_{outside} = t \times \gamma$
- light gets redshifted moving away from gravity source
  - $\lambda_{outside} = \lambda \times \gamma$
- moving masses create ripples in space-time (gravitational radiation)



Consequences of the Equivalence Principle

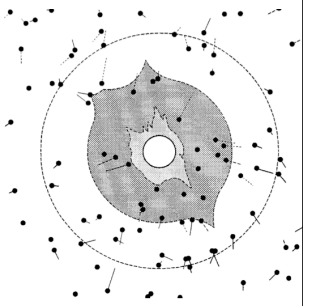
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- surface of the Sun: $d = 2.3 \times 10^5 \, r_s$ $\gamma = 1 + 2.14 \times 10^{-6}$
- " of a white dwarf: $d = 2.3 \times 10^3 \, r_s$ $\gamma = 1 + 2.14 \times 10^{-4}$
- " of a neutron star: $d = 3.3 \, r_s$ $\gamma = 1 + 0.20$
- as $d \rightarrow r_s$, $\gamma \rightarrow \infty$
  - *INFINITE* time dilation (time stops)
  - *INFINITE* redshift (wavelength increases to beyond radio)
  - *INFINITE* space curvature (no escape)
  - a **BLACK HOLE**

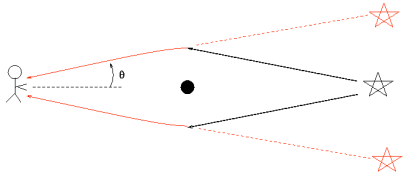


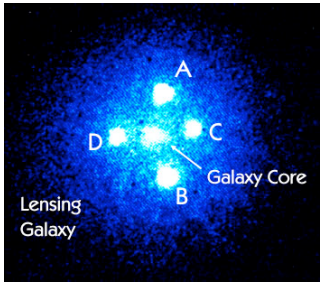
Tests of GR:

Bending of light by gravity



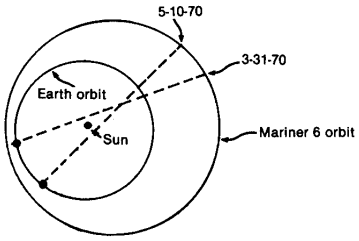
- displaced stars near limb of Sun - the 1919 and 1923 solar eclipses
- **Gravitational Lensing**

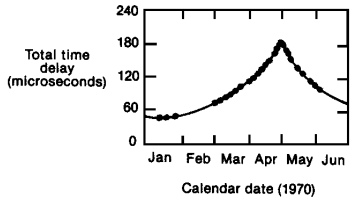


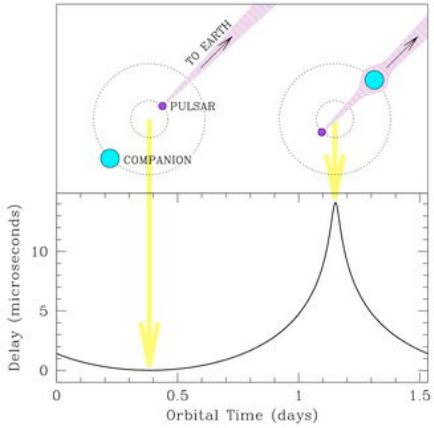


Shapiro Time Delay

- time for light to move along curved path is longer
  - Radar / spacecraft measurements within solar system
  - Binary pulsars



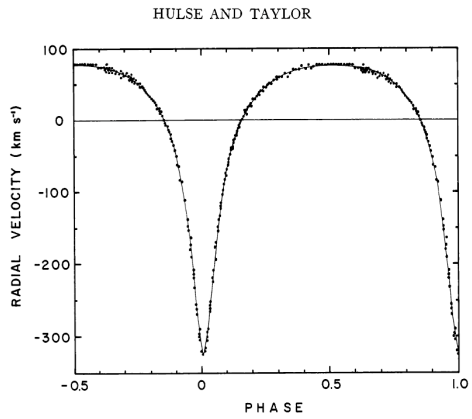




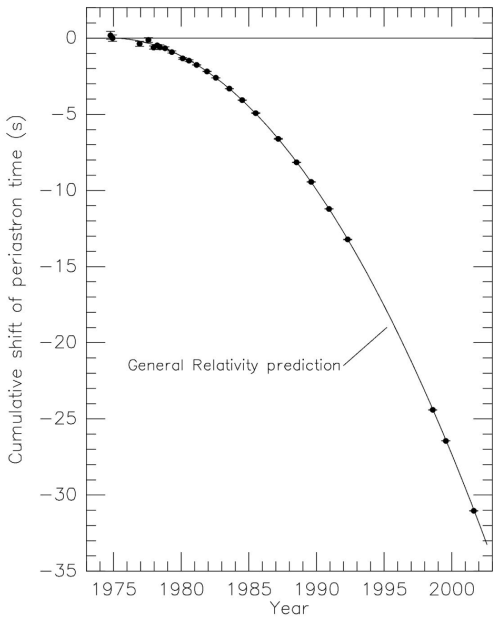
The Binary Pulsar

Hulse / Taylor 1993 Nobel Prize

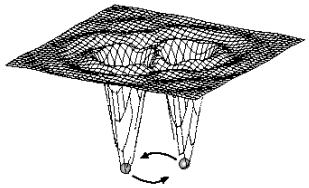
- Pulsar (clock!) in a binary
- orbit period = 7h 45m



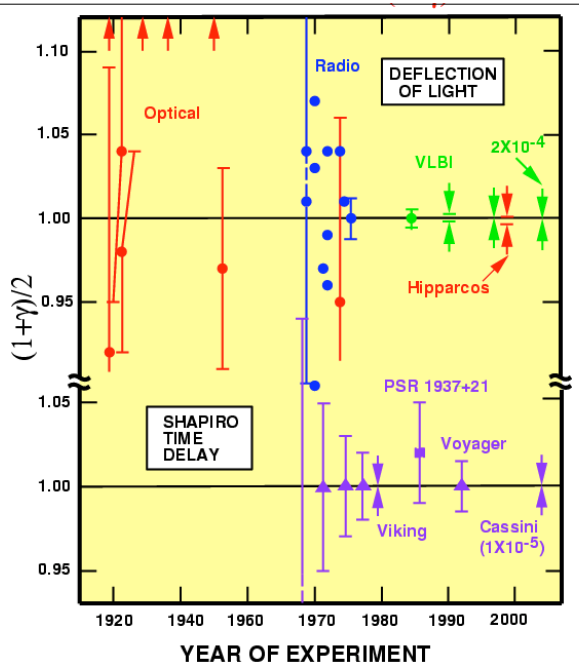
- neutron star - relatively large γ
  - large Shapiro time delay
  - large perihelion precession rate
  - orbital decay from gravitational radiation
  - (now) 30+ years of data



period change from **gravitational radiation** produces shift in periastron time



A binary system of compact massive objects rapidly orbiting each other produces ripples in spacetime.



http://relativity.livingreviews.org/open?pubNo=lrr-2006-3&page=articlesu7.html

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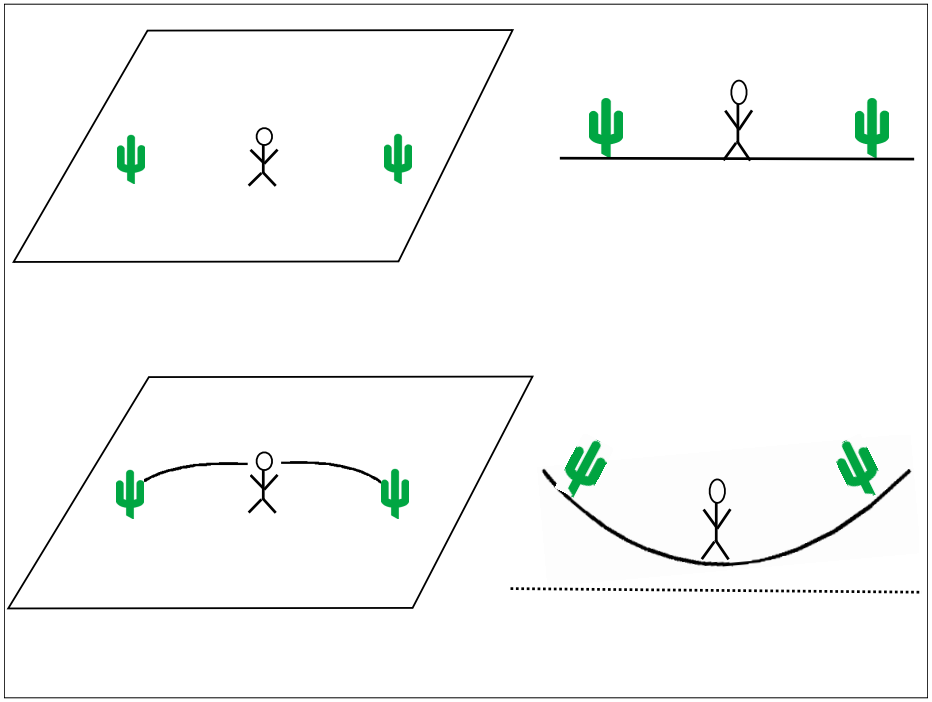
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  - *INFINITE* time dilation (time stops)
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  - *INFINITE* space curvature (no escape)
  - a **BLACK HOLE**

the "photon sphere"

- **event horizon**:
  - $v^2_{esc} = c^2 \sim 2GM/r$
  - **escape** velocity = c
- prior to (above) the event horizon:
  - $v^2_{orb} = c^2 \sim GM/r$
  - **orbital** velocity = c
  - $r_{photon} = 1.5 \times r_s$
- at this **photon sphere**
  - tangential light "orbits"
  - light directed down: spirals into BH
  - light directed upwards: spirals away

Approaching a black hole

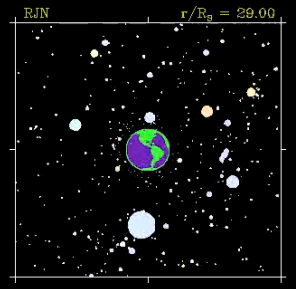
- pre-encounter (d > 1.5 r_s)
  - horizontally directed light curves downwards
  - perceived horizon bends *upwards*
    - the "bowl effect"
  - look "down" and see a bright ring at the photon sphere ("Einstein ring")



Approaching a black hole

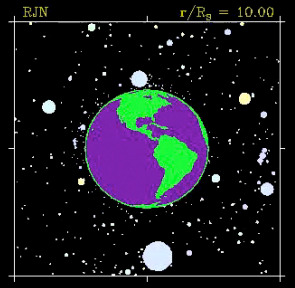
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  - horizontally directed light curves downwards
  - perceived horizon bends *upwards*
    - the "bowl effect"
  - look "down" and see a bright ring at the photon sphere ("Einstein ring")
- at the photon sphere
  - bowl effect very pronounced
  - straight ahead you see the back of your head!
- continue down (between P.S. and event horizon)
  - light *may* escape, depending on direction
  - escape cone (above) narrows on approach to r_s
  - outside of escape cone, view around corners

to a neutron star



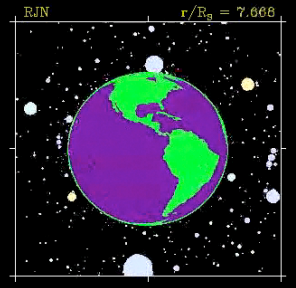
- "painted" with map of Earth
- orbit
- land
- look up
- orbit at surface

to a neutron star



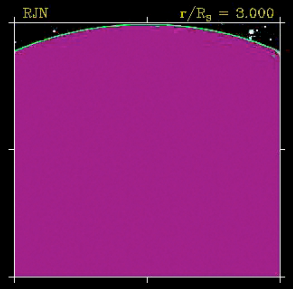
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to a neutron star



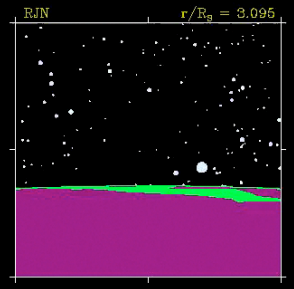
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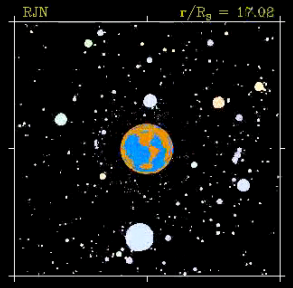
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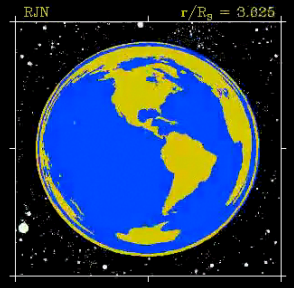
- "painted" with map of Earth
- orbit
- land
- look up
- orbit at surface

to an ultracompact star



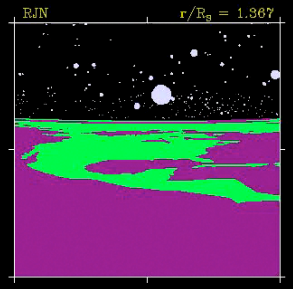
- "painted" with map of Earth
- orbit
- to photon sphere
- look up @ p.s.
- orbit
- to surface
- survey
- orbit

to an ultracompact star



- "painted" with map of Earth
- orbit
- to photon sphere
- look up @ p.s.
- orbit
- to surface
- survey
- orbit

to an ultracompact star



- "painted" with map of Earth
- orbit
- to photon sphere
- look up @ p.s.
- orbit
- to surface
- survey
- orbit

what lies beneath?

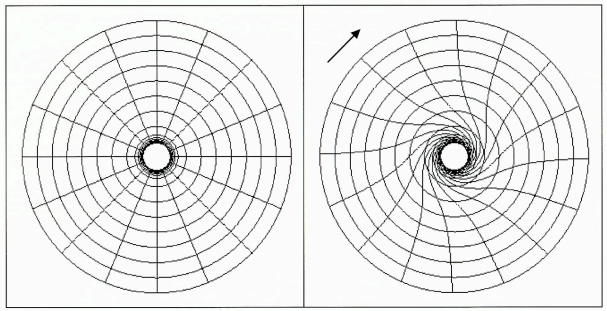
- non-rotating: **$d < r_s$**
  - fall @ speed of light to form a singularity
  - viewed from afar, collapse stops at event horizon

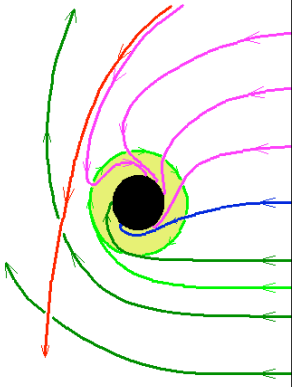
- **Cosmic Censorship**
  - time stops → loses meaning → is irrelevant!
  - cut off from the rest of the Universe (almost) forever

rotating black holes

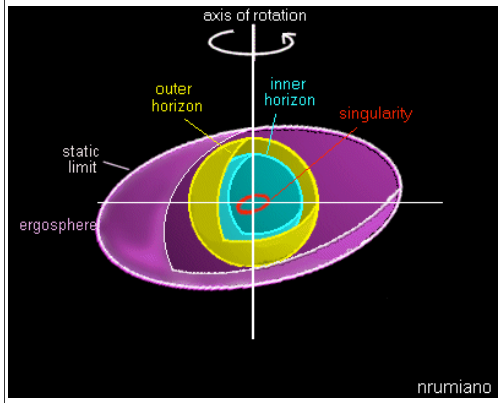
"everything" rotates

- rotation "drags" space-time along
- effect is to pull stuff in direction of BH spin
- extended capture region beyond r_s
  - "frame dragging"





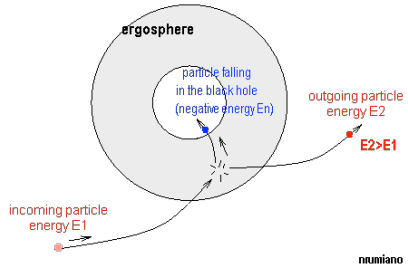
anatomy of a rotating (Kerr) black hole



- **static limit**
  - forced co-rotation
  - even light is forced to co-rotate w/ BH
- **outer horizon**
  - frame dragged faster than light
  - swap of time and space
  - singularity in time, not space
  - escape possible depending on initial entry
- **inner horizon**
  - return to normal time/ space

fast rotation ⇒ ring singularity

- nested "Event Horizons" surrounding the ring
- pass through ring to encounter exo-EH
- return to 3+1D Universe with more(!) energy
  - for some trajectories, anyway

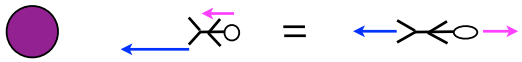


Black holes have no hair

- **Black holes have**
  - MASS (but no "surface")
  - SPIN
  - CHARGE
- **Black holes DO NOT have**
  - anitimatter / matter
  - color
  - smell
  - taste
  - texture
  - information
  - "hair"

Perils of visiting a black hole

- **Tidal forces**: differential pull of gravity
  - $F_{grav} \propto 1/d^2$ so $F_{tide} \propto 1/d^3$



- normal tides are tiny:
  - lunar tide = pillow on your head / 10^{12}

- $$\frac{stretch}{weight} = 10^7 \times height\,[m] \times \left[\frac{d}{R_{Sch}}\right]^{-3} \times \left[\frac{M}{M_{sun}}\right]^{-2}$$
  - Earth: s/w < 10^{-6}

$$\frac{stretch}{weight} = 10^7 \times height\,[m] \times \left[\frac{d}{R_{Sch}}\right]^{-3} \times \left[\frac{M}{M_{sun}}\right]^{-2}$$

- <u>Solar mass Black Hole:</u>
  - stretch = weight @ d = 3650 km (~1200 R_{sch})
  - " = 10 x weight @ d = 1690 km (~ 560 R_{sch})
  - **torn apart**
    - before getting anywhere near event horizon
    - before getting anywhere near NS surface
- NOTE: s/w $\propto$ 1/M : Bigger M = smaller stretch
  - S/W = 10 @ R_{sch} for M = 13,600 M_{sun}
  - S/W = 1 @ R_{sch} for M = 43,000 M_{sun}
  - can "easily" visit monster black holes

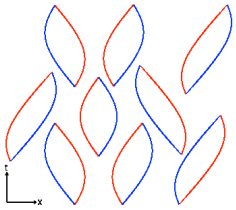
A visit to a BH

approach EH, send a regular beacon

- **You see**
  - normal time passage
  - accelerating downward
  - narrowing circle of outward visibility
  - blue stars, distorted sky
  - hole above closes as you cross EH
  - all views lead into BH
- **Companion sees**
  - beacon signals further and further apart
  - you falling slower
  - beacon weakens, reddens
  - evenutally frozen, reddened (no signals)

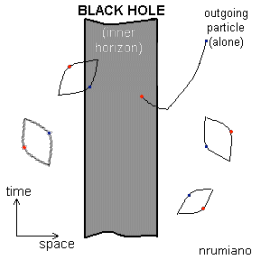
Black Holes don't live forever

- Heisenberg: $\Delta E \times \Delta t \geq$ (tiny #)
  - ΔE: accuracy
- E ≡ 0 in the vacuum?
  - no! only sure to $\Delta E > \hbar/\Delta t$
  - for very short time, temporary (large) E possible
  - "vacuum energy"
- virtual particles
  - can pop into existence briefly
  - matter / antimatter pairs
  - short life - antimatter particle soon annihilates his twin



Black Holes don't live forever

- virtual particles near a black hole
  - energy of a BH includes gravitational potential energy
  - produce virtual particle pair near (but outside) EH
  - if -E enters BH, +E escapes, BH Mass goes down
- "Hawking Radiation"
  - $L = 4\pi R^2 \sigma T^4$
    $= (\Delta mc^2)/t = 4\pi\,[3\text{km}(M/M_{sun})]^2 \sigma T^4$
- $$T_{BH,\,HR} = 2\times10^{11} K \left(\frac{M}{4\times10^{14} grams}\right)^{-1}$$
- $$t_{evap} = 10^{10} years \left(\frac{M}{4\times10^{14} grams}\right)^{3}$$
- for a solar mass black hole,
  - T = 4×10^{-8} K and t_{evap} = 1.25 x 10^{66} years!



Black Hole evaporation

(and mini-black holes)

- mini-BH as a power source
  - $4\times10^{14} c^2 = 3.6\times10^{35}$ erg = 1 L_{sun} for 2 minutes!
  - L = 4.1 x 10^{15} erg/s
    = 4.1×10^{8}W = 410 megaW! - a modest power plant!
  - but where do you put it?!
- "primordial" mini black holes
  - M < 4 x 10^{14} g → would have "popped" by now
  - M < 1.9 x 10^{14} g → " " " 10^9 yr after BB
- Low mass BH are HOT!
  - M ~ 6.0 x 10^{14} g → t_{evap} = 30 Gyr, $T_{HR} = 3 \times 10^{11}$K
  - if common could produce a gamma-ray background
  - none seen, so $M_{LMBH} < 10^{-8} M_{univ}$
- The Krennrich search for evaporating primordial BH