Some Radiation Theory

Why?

- The average temperature of a planet depends on how much energy a planet receives, and how it stores it.

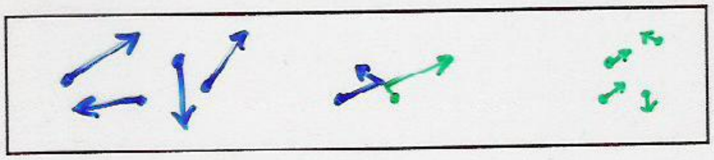
Variations, like seasons and climate, also depend on heat is stored and transported.

- We also need basic radiation theory to understand the temperature and luminosity of stars (including the sun).

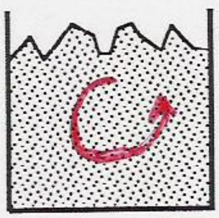
To begin...

Three ways to transport heat...

- **Conduction:** random molecular motion.



- **Convection:** larger scale circulations and turbulence



- **Radiation...** what is radiation? We need a little more theory.

Review: Basic Characteristics of EM. Radiation

I. Traveling wave characteristics:

Wave amplitude:

$$\vec{E} = \vec{E}_o \sin\left(\frac{2\pi}{\lambda}(x - ct)\right)$$

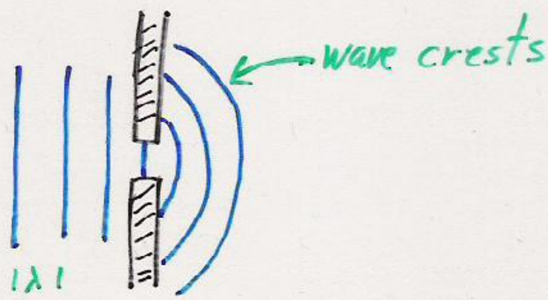
propagation speed

wavelength

The frequency and wavelength are related by the

dispersion relation: $\boxed{\lambda\nu = c}$ in vacuo.

Diffraction: waves "bend" around obstacles.

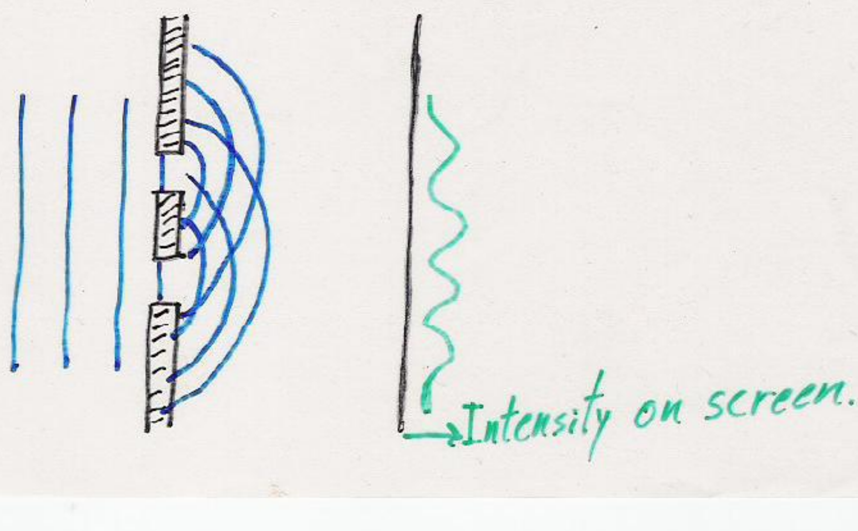


Interference: the amplitudes of overlapping waves add directly, but the brightness or intensity depends on the square of the sum.

$$I \propto \left[E_1 \sin\left(\frac{2\pi}{\lambda}(x_1 - ct)\right) + E_2 \sin\left(\frac{2\pi}{\lambda}(x_2 - ct)\right)\right]^2$$

(where this one-dimensional version of the eq. is used for simplicity).

E.g. the double-slit pattern,



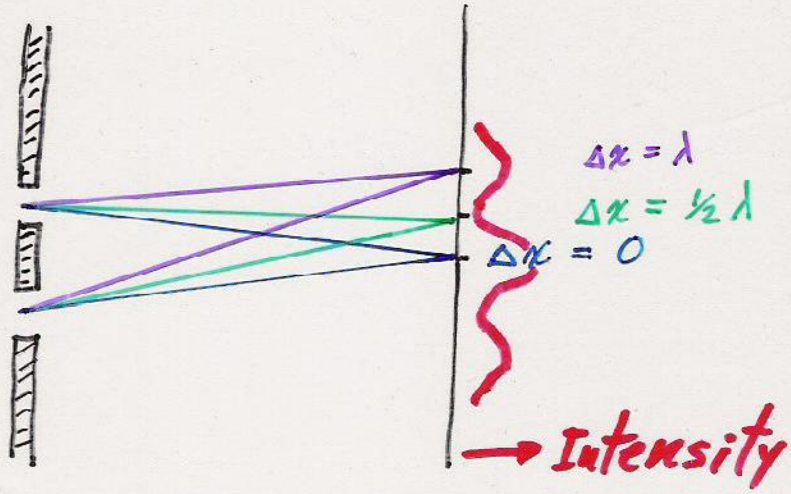
II. Light Rays, Geometric Optics

Reflection: Equal angles of incidence and reflection as measured from a normal to the surface. Allows the focussing of light.

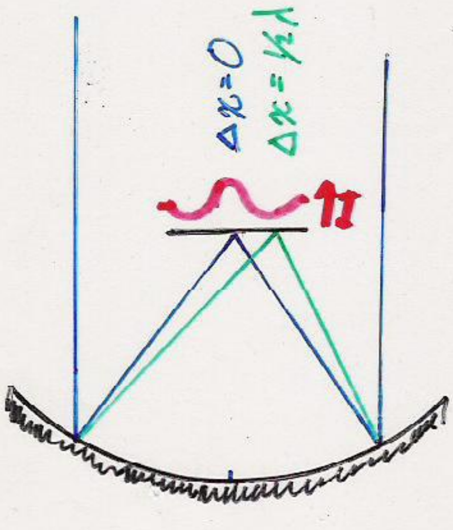
Refraction (dispersion): Generally light is bent as it passes through the boundary between two media. Dispersion results when the bending (or index of refraction) depends on wavelength λ.

Ray optics + waves help us better understand interference.....

2 slits:



Mirror reflection:



Diffraction pattern

III. Particle Aspect of Electromagnetic Radiation:

In addition to its wave nature, electromagnetic radiation also seems to come in discrete bundles of energy called "photons". The energy of a photon depends on the frequency of the radiation via Planck's law:

$$\boxed{\varepsilon = h\nu = hc/\lambda.}$$

This relation connects the wave-particle dual characteristics of em. radiation.

The particle aspect is most relevant at high energies, or very low light levels, i.e. where there are few photons.

The Doppler Effect

Often explained by an analogy to sound or water waves. See squeezed wave diagram below.

Full special relativistic expression for wavelength of light from a moving source is,

rest frame wavelength

$$\lambda/\lambda_0 = \left[\frac{1 + v/c}{1 - v/c}\right]^{1/2}$$

When $v/c \ll 1$, can approximate this by,

$$\lambda/\lambda_0 \approx (1 + \tfrac{1}{2} v/c)(1 - (-\tfrac{1}{2} v/c)) \simeq 1 + v/c$$

so,
$$\frac{\Delta\lambda}{\lambda_0} = \frac{\lambda - \lambda_0}{\lambda_0} = \lambda/\lambda_0 - 1 \simeq v/c$$

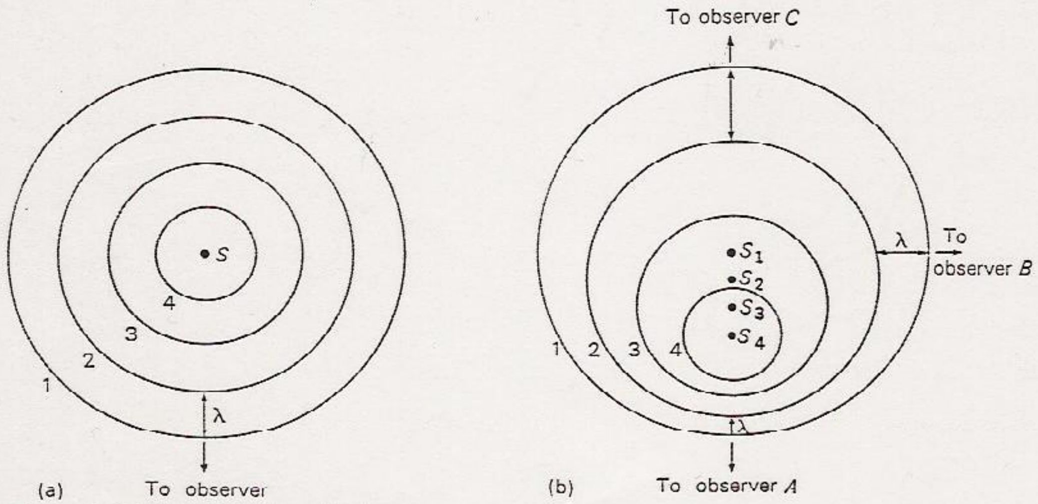


Figure 8.17 The Doppler effect.

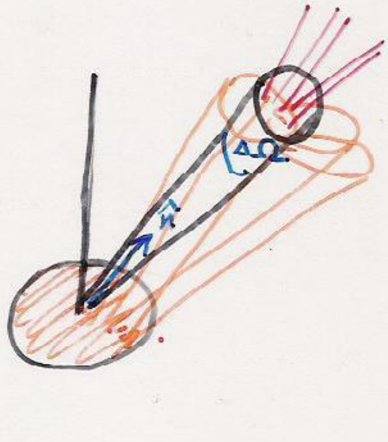
Abell, Morrison & Wolfe

Intensity and Flux

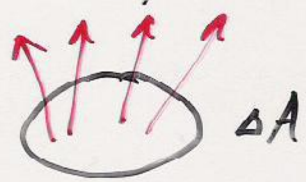
To describe the flow (and scattering) of radiation we need the concepts of intensity and flux, which quantify the idea of a directed density of radiation.

Monochromatic intensity - is the amount of energy in frequency interval $(\nu, \nu+\Delta\nu)$ flowing through a unit area, per unit time, into a cone of unit solid angle, in a given direction $\hat{n}$:

$$\vec{I}_\nu = \frac{\Delta E}{\Delta\nu\,\Delta A\,\Delta t\,\Delta\Omega}\,\hat{n} \qquad \frac{\text{ergs}}{\text{s}\cdot\text{cm}^2\cdot\text{Hz}\cdot\text{ster.}}$$



Monochromatic Flux is easier! Integrate intensity in solid angle over a hemisphere. Thus, flux is the energy in frequency range $(\nu, \nu + \Delta\nu)$ passing through a unit area per unit time into 'any' direction.



Alternately, the flux <u>received</u> at a detector is total incoming energy per unit time, etc., from any 'angle' (though generally from a single pt. source).

Thermal Radiation (Blackbody, Cavity Radiation)

The theory of the interaction of electromagnetic waves with solids, liquids, and gases began at about the same time as the "in vacuo" em. theory. In 1859 Kirchhoff summarized the work to date in 3 empirical rules.

Kirchhoff's Laws (not in book)

1. A solid, liquid or a gas under high pressure, when heated to incandescence will produce a continuous spectrum.

2. A gas under low pressure, but at a sufficiently high temperature, will give a spectrum of bright emission lines.

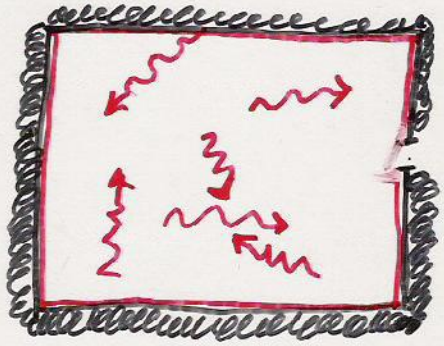
3. A gas at low pressure (and low temperature), lying between a hot continuum source and the observer, produces a number of dark, absorption lines on the continuous spectrum.

(Each chemical element has characteristic emission-absorption lines.)

We will see many examples later, when we study spectral types.

Continuous Thermal Emission

Consider the special case of an idealized equilibrium thermal radiator or Blackbody, e.g. a perfectly absorbing (and emitting) cavity.



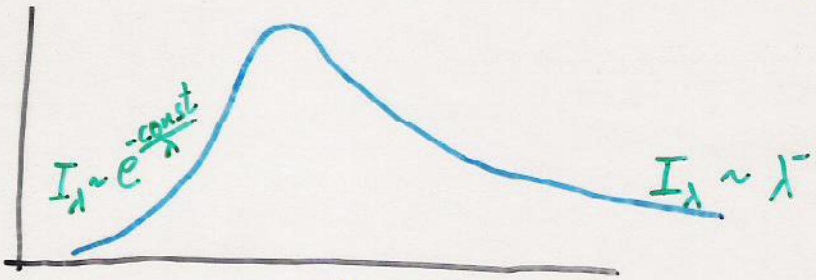
The radiation in the cavity is in equilibrium with the walls (which are painted black to absorb, not reflect radiation to facilitate equilibration).

Walls are insulated and heated slightly to compensate for losses through the peephole.

Properties of Thermal Radiation

(1) Thermal radiation has a continuous spectrum.

Intensity I_λ



$\lambda \longrightarrow$

Why? Radiation is constantly absorbed and reemitted by matter in the 'cavity' walls. Many emission processes involve collisions between atoms or electrons.

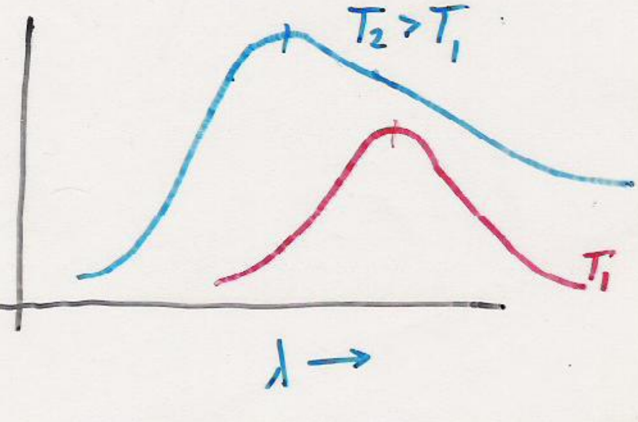
Photon energy depends on collision energy, and there is a wide dispersion of this.

(2) **Wien's law** the hotter the cavity, the bluer the peak wavelength

(and higher average energy per photon).

$$\lambda(cm) = \frac{0.29}{T(K)}$$

I_λ



$\lambda \longrightarrow$

③ Stefan-Boltzmann Law

when the cavity gets hotter the total energy increases as $\boxed{E = \sigma T^4}$

for "stars" we use

$$\frac{L}{L_\odot} = \frac{R^2}{R_\odot^2}\frac{T^4}{T_\odot^4}$$

$(E = L/4\pi R^2)$

④ Rayleigh-Jeans Law

At low frequencies $\boxed{I(\nu, T) \propto \nu^2.}$

Extrapolation of this law precipitated a crisis in physics.

All of these laws are contained within the Planck Equation for blackbody radiation:

$$I(\lambda) = \left(\frac{2hc^2}{\lambda^5}\right)\left[\frac{1}{e^{hc/\lambda kT} - 1}\right]$$

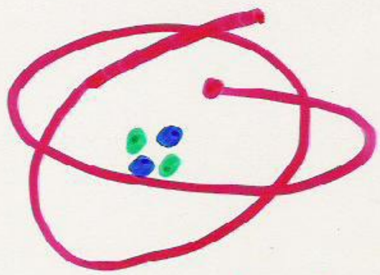
Why are we so interested in the "special case" of blackbody radiation? Stars are approximate BBs! The universe was once a BB!

Nonthermal Continuum Radiation

The simplicity and universality of blackbody radiation result from the requirement of equilibrium between radiation and matter. (The same is true of thermal equil., Boltzmann and spectral lines.) One variable T determines the blackbody spectrum. Many radiation processes in astrophysics produce a continuum in nonequilibrium settings, e.g. when the gas is not sufficiently opaque. Example – nonequilibrium recombination continuum. Example 2 – the radiation of charged particles moving in a magnetic field – Synchrotron Radiation.

Atoms and Spectral Lines

Atoms: small dense nucleus, p & n.
Usually number of p ≈ n.
Light electrons orbit nucleus like planets around, but complicated orbits.



In Bohr's model - orbits quantized.

Electrons stay in discrete, well-defined orbits or 'shells'.

① Orbits (shells) balance Coulomb attraction and centripetal force.

$$\frac{mv^2}{r} = \frac{(Ze)e}{r^2}$$

The discreteness comes from the assumption that orbital angular momentum is quantized.

$$mvr = \frac{nh}{2\pi} = n\hbar$$

② Note especially the existence of a lowest, or ground state orbit, $n=1$.

This doesn't exist classically, is intrinsically quantum.

③ The spacing between orbits gets smaller for large n.

$$\frac{m}{r}\left(\frac{n\hbar}{mr}\right)^2 = \frac{(n\hbar)^2}{mr^3} = \frac{Ze^2}{r^2}$$

substituting for v.

$$\therefore \quad r_n = \frac{(n\hbar)^2}{mZe^2}$$

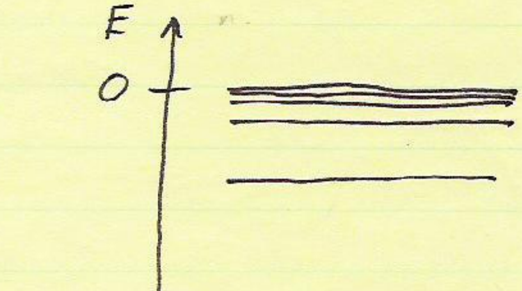
$$\Delta r = r_{n+1} - r_n = \left(\frac{\hbar^2}{mZe^2}\right)\left[\underbrace{(n+1)^2 - n^2}_{2n+1}\right]$$

$$\frac{\Delta r}{r_n} = \frac{2n+1}{n^2} \simeq \frac{2}{n}$$

In terms of orbital energy -

$$E_n = \tfrac{1}{2}mv^2 - \frac{Ze^2}{r} = -\frac{me^4Z^2}{2n^2\hbar^2}$$

$$\Delta E = E_{n+1} - E_n = -\frac{me^4Z^2}{2\hbar^2}\left[\frac{1}{(n+1)^2} - \frac{1}{n^2}\right]$$



(4) There exists a continuum, $E \geq 0$, where electrons are no longer bound.

(5) The structure of orbits is different for each element. The orbits are drawn closer in for elements with more protons, larger A.

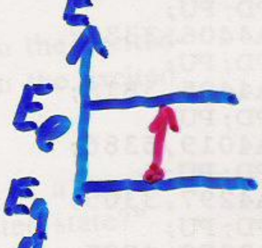
(6) Within shells there can be subshells (quantum numbers n, l, m, ...). The number of electrons in the outer shell determines the chemistry.

(7) The Pauli exclusion principle states that no more than 2 electrons can occupy a given orbit.

Line Radiation

Absorption: e^- lifted from ground state E_g to excited state E_e by absorbing a photon of energy

$$\varepsilon = h\nu = E_e - E_g$$



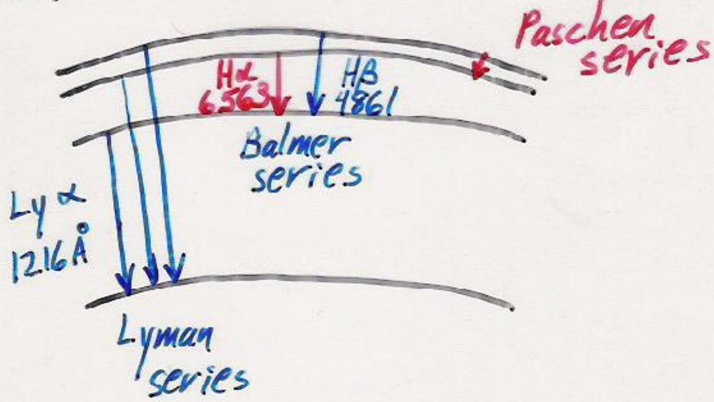
Generally, $h\nu_{nm} = E_n - E_m$

for $n > m$

Emission: e^- spontaneously drops to lower orbits, until it reaches ground state.

Photon emitted each time.

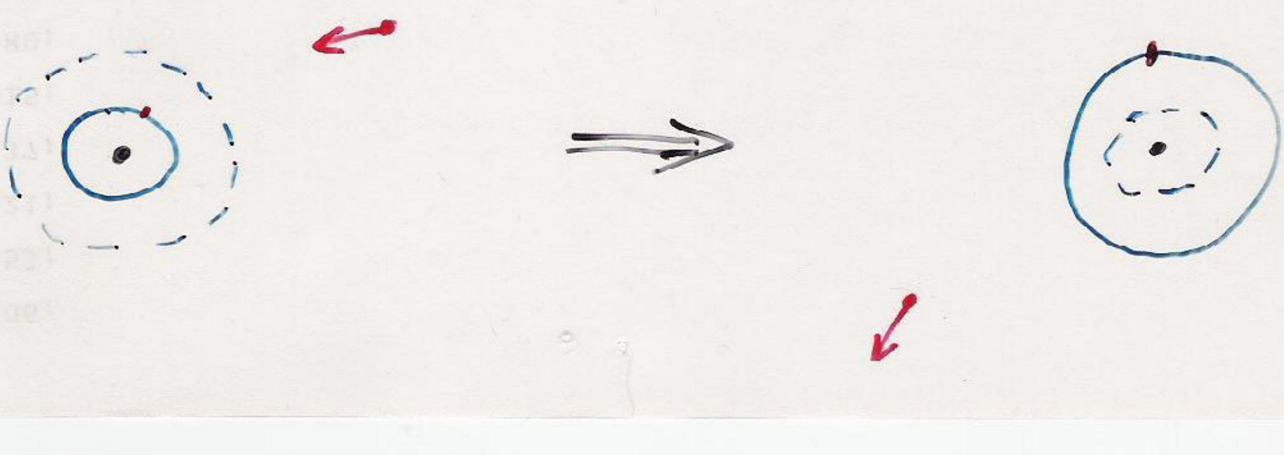
The most important example in astronomy is the hydrogen atom



Excitation and Ionization

At low temperatures and with no external disturbances atoms like to live in the ground state. Two ways to excite electrons to higher energy levels.

① Collisions



Some of the relative kinetic energy of the collision is used to boost the electron.

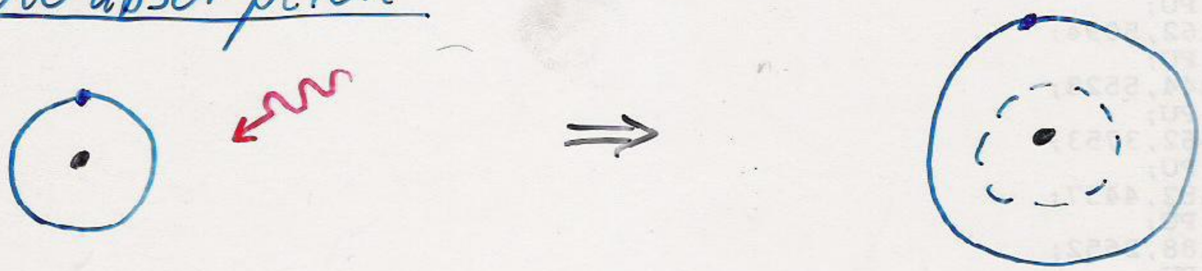
Low T (Maxwellian dist.) $\Longleftrightarrow$ few fast collisions.

High T " $\Longleftrightarrow$ many fast collisions.

If enough energy is imparted to the electron in the collision it may be pried free.

Collisional ionization

(2) Photo absorption



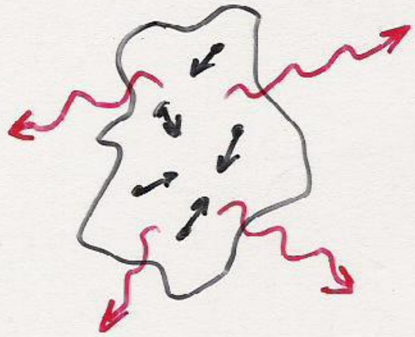
Electron absorbs photon with energy equal to the difference between levels.

For photoionization need $E_{photon} > |E_{bind}|$.

We can now understand Kirchhoff's laws from a microscopic point of view.

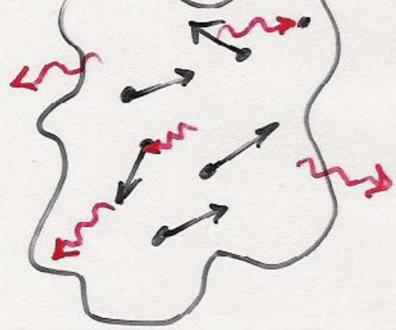
② Low density, non-opaque gas that is hot enough to excite $n = 2, 3, \ldots$ levels efficiently

$\Rightarrow$ emission lines



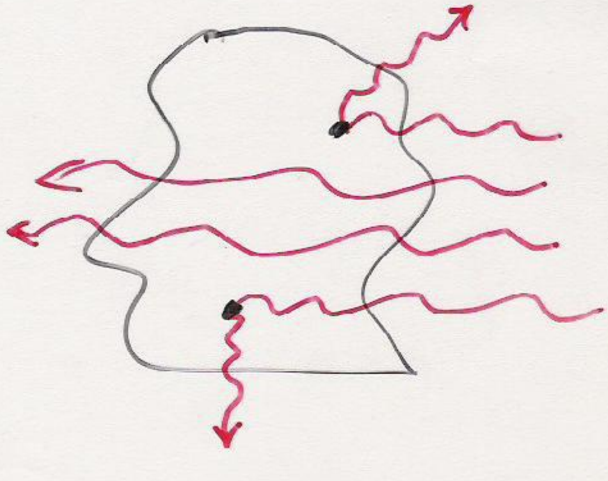
① Very hot, opaque gas, with much ionization, excitation, and recombination

$\Rightarrow$ continuum emission.



③ Cold gas: photoabsorption

$\Rightarrow$ re-emission into random directions + absorption lines in forward direction.



Optical Depth and the Transfer of Radiation

To begin, the photon scattering probability is formalized by defining the **optical depth.** The differential optical depth and the differential scattering probability are essentially the same,

$$\Delta\tau = n\,\sigma_\nu\,\Delta l,$$

where optical depth is conventionally measured from the outside-in.

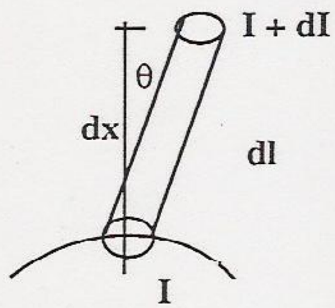
Integral form: $$\tau_\nu = \int_{path} n(x)\,\sigma_\nu dx$$

We also have,

$$\Delta\tau_\nu = \frac{\rho}{\langle m\rangle}\,\sigma_\nu\,\Delta l = k_\nu\,\rho\,\Delta l,$$

where <m> is the mean particle mass, and k_ν is the opacity in units of m^2/kg.

Now consider the radiation flux passing through a small cylinder in the Sun's atmosphere,



A fraction of the radiation is lost in the cylinder due to scattering. This equals the scattering probability time I_ν. Then assuming that there are no sources of emission,

$$\Delta I_\nu = I_\nu \frac{\Delta\tau}{\cos\theta} = k_\nu \rho \left(\frac{\Delta x}{\cos\theta} \right) I_\nu .$$

or, $$\cos\theta \frac{dI_\nu}{d\tau_\nu} = I_\nu .$$

This can be directly integrated to get,

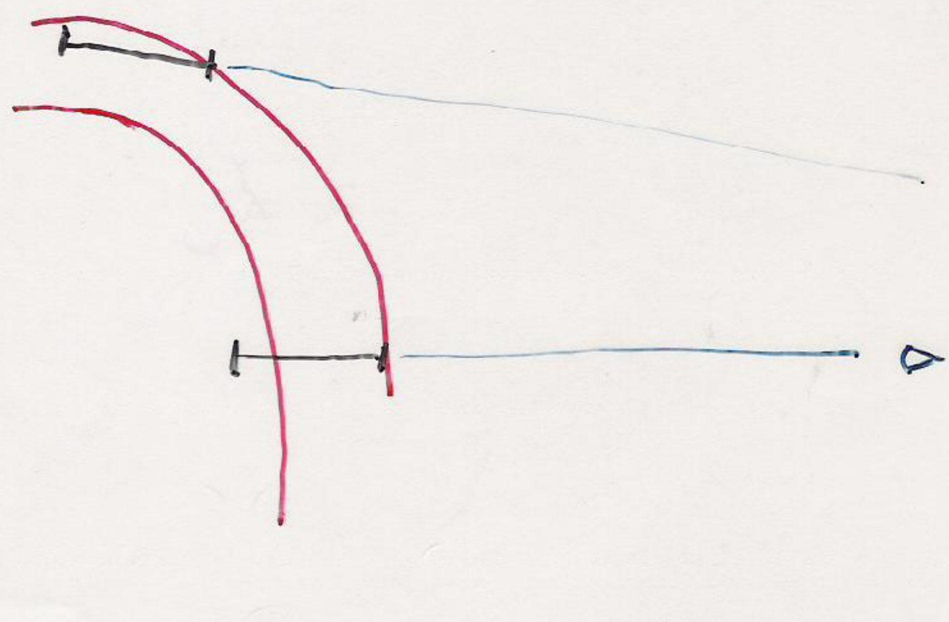
$$I_\nu(\tau_\nu) = I_\nu^o \exp(-\tau_\nu / \cos\theta).$$

The second (exp. factor) is the integral scattering probability.

Analogy the intensity falls off with τ into a stellar atmosphere just like pressure falls with height in an isothermal planetary atmosphere.

Limb Darkening

Can see down to an optical depth of about $2/3$ (because $e^{-2/3} \simeq \frac{1}{2}$) anywhere on the face of sun. Assume $n\sigma \simeq$ const., then $\Delta\tau = 2/3$ corresponds to a fixed path Δl. This serves as a constant 'meter' stick for 'dipping' into the sun along the line of sight.



A Little ~~Back to~~ Radiative Transfer

When there is emission as well as absorption/scattering in a volume of atmosphere, transfer eq. becomes,

$$\cos\theta \frac{dI_\nu}{d\tau} = I_\nu - S_\nu$$

with $S_\nu = \frac{\eta_\nu}{k_\nu \rho}$, the "source function",

and η_ν = the emission coefficient (in units of I_ν/ℓ).

Formal solution of transfer eq.:

$$I_\nu(\tau) = I_\nu(0)\, e^{-(\tau_2-\tau_1)/\cos\theta} + \frac{1}{\cos\theta}\int_{\tau_1}^{\tau_2} S_\nu\, e^{-(\tau-\tau_1)/\cos\theta}\, d\tau$$

- When $S_\nu \approx 0$, we have the previous, <u>optically thin</u>, $\tau \ll 1$, solution. (Good for planets.)

- When $\cos\theta = 1$, and we have a slab with $S_\nu \approx$ const. and no flux $I_\nu(0)$ getting directly through, we have

$$I_\nu = S_\nu\,(1 + e^{-\tau}), \quad (\text{with } \tau_1 = 0,\ \tau_2 = \tau),$$

or $I_\nu = S_\nu$, the blackbody intensity for $\tau \gg 1$.
(Stars!)